An Example Using the Intermediate Value Theorem

Intermediate Value Theorem (I.V.T.)

If f is a **continuous** function on the **closed** interval [a,b] and N is any number between $f(a)$ and $f(b)$ then there exists a number $c \in (a,b)$ such that $f(c) = N$.

Problem: Show that the cube root of 3 (three) exists.

Before beginning, you must realize exactly what the cube root of 3 means! The symbol $\sqrt[3]{3}$ stands for a number c that has the special property that when you cube it you obtain 3. That is: $\boxed{c = \sqrt[3]{3} \text{ if and only if } c^3 = 3}$

Solution:

Let $f(x) = x^3$. Now, $f(1) = 1$ and $f(2) = 8$. Since f is continuous on [1,2] (because f is a polynomial) and $f(1) < 3 < f(2)$ it follows by the *Intermediate Value Theorem* that there exists a number c, $c \in (1,2)$, such that $f(c) = 3$. That is, $c^3 = 3$ and it follows that c is the cube root of three (i.e. $\sqrt[3]{3}$ exists).

Q.E.D.