$\frac{dy}{dx}, \frac{dx}{dy}$ and $\frac{d}{dx}\left(f^{-1}(x)\right)$

Recall: If *f* is invertible then $y = f^{-1}(x)$ (*) if and only if $f(y) = x$ (**).

If we differentiate (**) with respect to *x* we obtain:

$$\frac{d}{dx}\left(f(y)\right) = \frac{d}{dx}(x)$$

$$f'(y)\frac{dy}{dx} = 1 \qquad \text{(since } y \text{ is a function of } x\text{)}$$

$$\frac{dy}{dx} = \frac{1}{f'(y)} = \frac{1}{f'(f^{-1}(x))}$$

so $\boxed{\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))}}$. In words, this says that the **derivative of an inverse is the reciprocal of the derivative of the original function *evaluated* at** $f^{-1}(x)$

Now, differentiate (**) with respect to *y* to obtain:

$$\frac{d}{dy}\left(f(y)\right) = \frac{dx}{dy}$$

$$f'(y) = \frac{dx}{dy}$$

Putting the two together yields: $\boxed{\frac{dy}{dx} = \frac{1}{f'(y)} = \frac{1}{\frac{dx}{dy}}}$ assuming, of course, that the individual derivatives exist and that $\frac{dx}{dy} \neq 0$.

Similarly, you can obtain that $\boxed{\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}}$ with the appropriate qualifications on the derivatives.