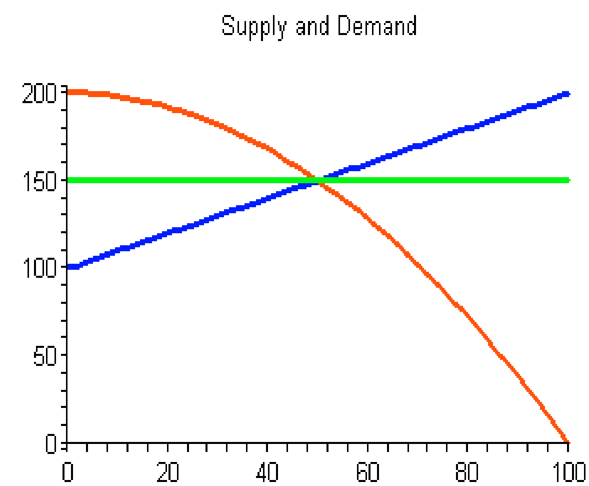
Consumer & Producer Surplus

The **supply function**, $p = s(x)$, for a commodity, specifies the relationship between price p and the number of units x of the commodity that would be supplied by manufacturers. So $s(x_k) = p_k$ means that if the market price is p_k then the manufacturer or producer will supply x_k units.

The **demand function**, $p = p(x)$, is the price that must be set if x units are to be sold. So $p(x_k) = p_k$ means that if the seller's price is p_k he/she will sell x_k units.

Typically, the supply function is an increasing function (as the market price rises, the manufacturers will want to supply more units) while the demand function is a decreasing function (as the price increases, sales drop off).

Example: Suppose $p(x) = 200 - 0.02x^2$ and $s(x) = 100 + x$ are the demand and supply functions for a certain commodity. Sketch the curves and find the **equilibrium point**, the point at which supply equals demand.

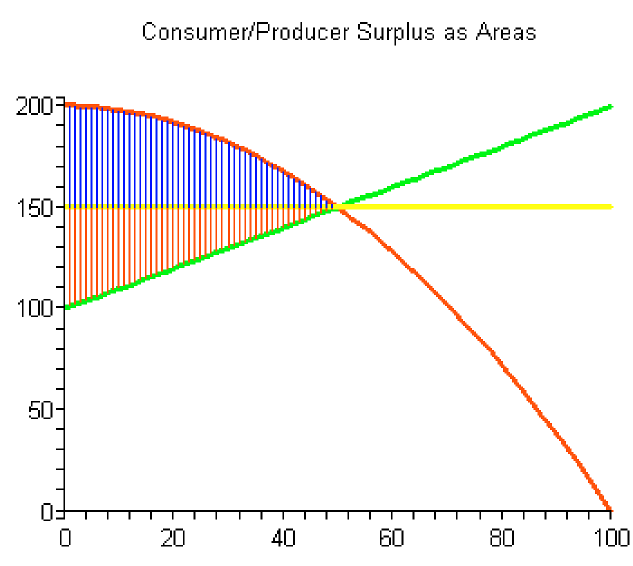


Solution: A computer generated plot of the two functions is supplied. They are plotted on their physical domain: the domain that makes sense for the physical problem. The red is demand and the blue is supply with a horizontal line drawn through the equilibrium point.

To find the equilibrium point you solve the equation $p(x) = s(x)$ for x:

$$200 - 0.02x^2 = 100 + x$$
$$0.02x^2 + x - 100 = 0$$

The quadratic formula can then be used to yield the result $x = 50, -100$ but the negative result is discarded for obvious reasons. To find the equilibrium price simply "plug in" to get $p(50) = s(50) = 150$.



Producer Surplus

There are two areas that are of interest in the supply-demand situation. The first one to consider is the area between the equilibrium and the supply curves from zero to the equilibrium x-coordinate. It is the red-shaded region in the plot to the right. If the co-ordinates of the equilibrium point are (x_0, p_0), then this area is calculated as $\int_0^{x_0} (p_0 - s(x))\,dx$. To see what this area represents is more difficult and requires a close look at the definition of the integral as a Reimann Sum: $\int_0^{x_0} (p_0 - s(x))\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} (p_0 - s(x_i))\Delta x$. To see what this represents, think of a representative rectangle above the interval $[x_{i-1}, x_i]$. The area of the rectangle is $(p_0 - s(x_i))\Delta x = p_0\Delta x - s(x_i)\Delta x$. Δx is the number of units being produced and sold during this interval. Since p_0 is what the market is paying, $\$p_0\Delta x$ is what the manufacturer will receive for the units. However, the manufacturer *would* have been satisfied with $\$s(x_i)\Delta x$ for these units (that is the meaning of the supply curve). It follows then that the quantity $(p_0 - s(x_i))\Delta x$ is actually **excess profit** for the producer – profit over and above with what he/she would have been willing to live. It follows that the original integral, $\int_0^{x_0} (p_0 - s(x))\,dx$, represents the total excess profit to the producer as a result of the difference between **market price** and what the producer would have been willing to accept.

If you were a government it would be worth your while to hire some hot shot econometrists – experts at measuring various economic indices – to figure out (estimate) the supply curves for various industries and use the producer surplus as a guide (amongst other things) to an intelligent taxation policy: a "healthy" producer surplus for an industry would indicate a high capacity to absorb increased taxes without undue strain on producers (which might cause them to look elsewhere).

<u>Consumer Surplus</u>

The other area of interest in the diagram is the area under the demand curve and above the equilibrium price (the blue area in the sketch): $\int_0^{x_0} (p(x) - p_0)\,dx$. To see what this represents, consider the definition: $\int_0^{x_0} (p(x) - p_0)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} (p(x_i) - p_0)\Delta x$.

The quantity $(p(x_i) - p_0)\Delta x = p(x_i)\Delta x - p_o\Delta x$ is the difference between what consumers *would have been willing to pay* for Δx units between x_{i-1} and x_i and what they *actually paid* due to the prevailing market price of p_0. Hence, $(p(x_i) - p_0)\Delta x = p(x_i)\Delta x - p_o\Delta x$ represents a (theoretical) saving to the consumers. (Imagine that you would be willing to pay $600 for your dream machine but when you went shopping you found it for only $400! You would interpret this as a saving and might even consider spending the $200 that you "saved" on a related item: a dream stand for your dream machine.)

The integral, $\int_0^{x_0} \left(p(x) - p_0 \right) dx$, is then measuring the total savings realized by consumers as a result of the discrepancy between market price and "willingness to spend." It could be that a product, or group of products or services, with a healthy consumer surplus indicates an area for business to fruitfully expand into – to take advantage of all those consumer savings.

Example: Calculate the producer and consumer surplus for a commodity which has demand and supply curves $p(x) = 200 - 0.02x^2$ and $s(x) = 100 + x$. Base your calculations on the equilibrium price.

Solution: From the first example we know that $p(x) = s(x)$ if $x = 50$ and $p_0 = 150$. The producer surplus is therefor $\int_0^{50} \left(p_0 - s(x) \right) dx = \int_0^{50} \left(150 - (100 + x) \right) dx = 1250$. The actual dollar amount this represents would depend on the units of the original problem.

The consumer surplus will be $\int_0^{50} \left(200 - 0.02x^2 - 150 \right) dx \approx 1666.67$ and the actual dollar amount this represents will again depend upon the units in the original problem.