Some Marginal Analysis

Suppose that $C(x)$ is a function that specifies the cost of producing *x* units of some commodity. If *C* is differentiable (i.e. has a derivative) then, by definition, $C'(x) = \lim_{h\to 0} \frac{C(x+h)-C(x)}{h}$ and is called the **marginal cost** of *x* units. If the cost is measured in dollars then the units of $C'(x)$ are "dollars per unit" and C' can be thought of as the rate of change of costs with respect to the number of units and measures the rate at which production costs are rising or falling as production levels vary.

Now, if the *h* is "small" in the limit definition then you can say that $C'(x) \approx \frac{C(x+h)-C(x)}{h}$. In the "real world" of commerce, the smallest *h* that makes physical sense is one (since you don't normally actually produce fractional units of a commodity). Therefor, it makes sense to consider $C'(x) \approx \frac{C(x+1)-C(x)}{1} = C(x+1)-C(x)$. In other words, the marginal cost of producing *x* units of a commodity can be reasonably be thought of as the *additional* cost incurred in producing *one more unit* (the $(x+1)^{th}$ unit). This is often how businesses calculate their marginal cost – by noting the extra expenses incurred by producing just one more unit.

As an example, consider a company that figures its cost of producing *x* units is given by $C(x) = 10,000 + 5x + 0.01x^2$ dollars. The marginal cost function would then be $C'(x) = 5 + 0.02x$ and the marginal cost at a production level of 500 units would be $C'(500) = \$15/unit$. You might interpret this by saying, "At a production level of 500 units, production costs are rising at the rate of $15 per unit." It is interesting to note that $C(501) - C(500) = \$15.01$ and so you can see that $C'(500) \approx C(501) - C(500)$ and that the estimate is, in fact, a very good one.

Similar statements and definitions and estimating procedures can be made for marginal revenue, marginal profit and marginal demand.