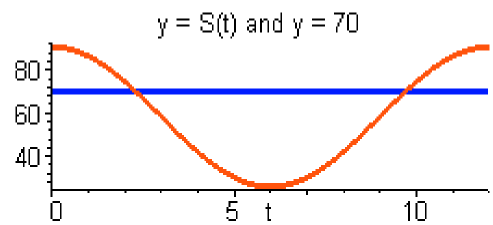
A Sales Problem Involving Trigonometry

Suppose a company that manufactures a product for which the demand is seasonal has determined that the number of units (in the thousands) it can sell during the t^{th} month (where t = 1 corresponds to January) is modeled by $S(t) = 58.3 + 32.5\cos\left(\frac{\pi t}{6}\right)$.

a) Use a graph to approximate when in the year the sales are 70,000 units/month.
b) Find the times correctly to one decimal place.
c) Determine the rate at which sales are increasing or decreasing during February (i.e. when t = 2).

Solution:

a) To visualize the situation, look at the following graph of $y = 58.3 + 32.5\cos\left(\frac{\pi}{6}t\right)$ and $y = 70$. From the graph, it "looks like" sales will be at 70 thousand units when t is about 2 and 10 which would mean in February and October.



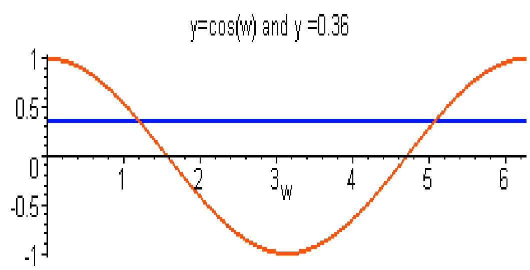
b) To find the times precisely, you have to solve the equation:

$$58.3 + 32.5\cos\left(\frac{\pi}{6}t\right) = 70$$

$$\cos\left(\frac{\pi}{6}t\right) = \frac{70 - 58.3}{32.5} = 0.36$$

To make the above equation "friendlier" we can let $w = \frac{\pi}{6}t$ and first solve $\cos w = 0.36$

Consider the accompanying sketch which shows one period of $y = \cos w$ together with $y = 0.36$.



The **first solution** is $w_1 = \cos^{-1}(0.36) \approx 1.2025$ (make sure your calculator is in **radian mode**).

From knowing about $y = \cos(w)$ we know that cosine is positive again Quadrant IV. Therefor, there is a **second** solution in Quadrant IV. It is the angle in Quadrant IV whose **reference angle** is $\cos^{-1}(0.36)$ (check out your "unit circle") which would be $2\pi - \cos^{-1}(0.36) \approx 5.08$. The two solutions look "reasonable" in terms of the graph above of $y = \cos w$ and $y = 0.36$

Finally, $w = \frac{\pi t}{6}$, so we have to solve $\frac{\pi t}{6} = \cos^{-1}(0.36)$ for *t* and obtain $t_1 = \frac{6}{\pi}\cos^{-1}(0.36) \approx 2.3$ (using a calculator **in radian <u>mode</u>**) and 2.3 corresponds to about February 9th.

For the second solution, t_2, we get:

$$\frac{\pi t_2}{6} = 2\pi - \cos^{-1}(0.36) \text{ to get}$$

$$t_2 = \frac{6}{\pi}\left[2\pi - \cos^{-1}(0.36)\right] \approx 9.7$$

(9.7 corresponds to about September 21.)

b) To find the rate at which sales are increasing in February we find the derivative of the sales function and "plug in" t = 2:

$$S'(t) = -32.5\sin\left(\frac{\pi t}{6}\right)\cdot\frac{\pi}{6} \text{ so}$$

$$S'(2) = -32.5\sin\left(\frac{\pi}{3}\right)\cdot\frac{\pi}{6}$$

$$= -32.5\cdot\frac{\sqrt{3}}{3}\cdot\frac{\pi}{6}$$

$$\approx -15$$

It follows that sales are **<u>decreasing</u>** (since the derivative is **negative**) at the rate of about 15,000 units per month in February.