

Total Change and the Definite Integral

Notice: $\int_a^b f'(x)dx = f(x)\Big|_a^b = f(b) - f(a)$ and $f(b) - f(a)$ is the total **change** in

the function f as x changes from a to b . In other words, the definite integral of a function which is a **rate of change** (i.e. a derivative) represents the total change in the original function.

Example: Suppose that a company has determined that its **marginal profit** function is $P'(q) = -0.0005q + 12.2$ where q is the number of units sold. Determine how profits will change if sales are increased to 110 units from a level of 100 units.

Solution: The resulting change in profits is:

$$\begin{aligned} & P(110) - P(100) \\ &= \int_{100}^{110} P'(q) dq = \int_{100}^{110} (-0.0005q + 12.2) dq \\ &= \left(-0.00025q^2 + 12.2q \right) \Big|_{100}^{110} \\ &\approx \$121 \end{aligned}$$

It follows that profits will increase by \$121 if sales are increased from 100 to 110 units.