Fundamental Theorem of Calculus

Part I: If f is continuous on [a,b] and $g(x)=\int_a^x f(t)dt$ for $a \le x \le b$ then g is differentiable on (a,b) and $g'(x)=f(x)$. That is, g is an antiderivative of f on [a,b].

<u>Proof</u>: See class notes or the text book.

Part II: If f is continuous on [a,b] and F is any antiderivative of f then $\int_a^b f(x)dx = F(a)-F(b)$.

<u>Proof</u>: Suppose that f is continuous on [a,b] and that F is an antiderivative of f. Let $g(x)=\int_a^x f(t)dt$ for $\mathrm{a} \le \mathrm{x} \le \mathrm{b}$. By Part I, g is also an antiderivative of f. It follows that:

$g(x)=F(x)+C$ (since any two antiderivatives of f can differ by at most a constant)

$g(a)=F(a)+C$ but $g(a)=\int_a^a f(t)dt=0$ so we get that

$C=-F(a)$.

Therefore: $g(b)=F(b)+C=F(b)-F(a)$ (but $g(b)=\int_a^b f(t)dt$)

And so: $\int_a^b f(x)dx = F(b)-F(a)$ (recall, the t is just a “dummy” variable).

Q.E.D.