Solving a Linear System Using Matrices and A^{-1}

Consider the matrices $A = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$. We showed in class that $A \cdot C = C \cdot A = I$ where I is the 2 x 2 identity matrix. This means that $C = A^{-1}$.

Now, consider the linear system of equations $\begin{cases} 2x - 5y = 1 \\ -x + 3y = -2 \end{cases}$. You can easily check, using the definition of matrix multiplication, that the matrix equation $\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is equivalent to the linear system and can be written simply as $AX = B$ where $A = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. We can then proceed as follows:

$$\begin{aligned} AX &= B \\ C\left(AX\right) &= CB \\ \left(CA\right)X &= CB \\ IX &= CB \\ X &= CB \end{aligned}$$

Doing the multiplication gives that $CB = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \end{pmatrix}$ and from this we get that $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \end{pmatrix}$ so $x = -7, y = -3$. Substituting these values into the system verifies their correctness.